Received: *June 28, 1999*, Accepted: *October 18, 1999*

# Vector mesons in the extended chiral quark model

**Alexander A. Andrianov*** **and Domenec Espriu**

*Departament d'Estructura i Constituents de la Matèria,*
*Universitat de Barcelona,*
*Diagonal, 647, 08028 Barcelona, Spain,*
*E-mail:* andrian@ecm.ub.es espriu@ecm.ub.es

Abstract: We extend our previous formulation of low-energy QCD in terms of an effective lagrangean containing operators of dimensionality $d \leq 6$ constructed with pseudoscalars and quark fields, describing physics below the scale of chiral symmetry breaking. We include in this paper the vector and axial-vector channels. We follow closely the Extended Chiral Quark Model approach and consistently work in the large-$N_c$ and leading log approximation and take into account the constraints from chiral symmetry and chiral symmetry restoration. The optimal fit of all parameters gives further support to a heavy scalar meson with a mass $\sim 1\,\mathrm{GeV}$ and a value of the axial pion-quark coupling constant $g_A \simeq 0.55$ to $0.66$, depending on some assumptions concerning the Weinberg sum rules.

Keywords: QCD, Phenomenological Models, Chiral Lagrangians.

*Permanent address: Department of Theoretical Physics, Sankt-Petersburg State University, 198904 Sankt-Petersburg, Russia.

JHEP10(1999)022

## Contents





## 1. Introduction of the extended chiral quark model

The purpose of this paper is to analyze the physics of low-lying scalar, pseudoscalar, vector and axial-vector mesons in the framework of the Extended Chiral Quark Model (ECQM) which was introduced in [1]. We shall see that this model of low-energy QCD is sufficiently general and robust to allow for a good description of the vector and axial-vector channels when the model introduced in [1] is suitably modified to make room for spin 1 resonances. On the other hand, the addition of vector and axial-vector resonances constraints the parameters of ECQM substantially if experimental data are to be fit consistently. Special emphasis is put on the axial pion-quark coupling constant[2] $g_A$ whose value happens to be crucial to obtain phenomenologically acceptable values of meson masses and coupling constants. This effective coupling was taken as a free input in the first version of the ECQM.

In a sense the ECQM lagrangean $\mathcal{L}_{ECQM}$ systematically extends both the Chiral Quark (CQM) [2, 3] and Nambu-Jona-Lasinio (NJL) models [4] (see reviews [5]–[10] and references therein) and is based on the non-linear realization of chiral symmetry.[1] Its inspiration is drawn from Wilson's renormalization-group ideas as well as the general principles of locality and gauge and chiral symmetry. The basic idea is to use the degrees of freedom which are relevant at each energy scale. It is therefore built

[1] There exist also extensions based on linear realization of chiral symmetry [11, 12, 13].

in terms of colored current quark fields $\bar{q}_i(x), q_i(x)$ with momenta restricted to be below the chiral symmetry breaking (CSB) scale $\Lambda_{CSB} \sim 1.3\,\mathrm{GeV}$, and colorless chiral fields $U(x) = \exp(i\pi(x)/F_0)$ which are SU($N_F$) matrices with generators $\pi \equiv \pi^a T^a$, $a = 1, \ldots, N_F^2 - 1$, and which, naturally, only appear below $\Lambda_{CSB}$. The quarks can be then endowed with a relatively large 'constituent' mass without manifestly breaking chiral symmetry. The information on modes with frequencies larger than $\Lambda_{CSB}$ is contained in the coefficients of the effective lagrangean, and used as boundary conditions (at scale $\Lambda_{CSB}$) of the renormalization-group equations. In addition, some residual gluon interactions remain below $\Lambda_{CSB}$. These residual gluon interactions (much diminished after the explicit inclusion of pions and other resonances) make the model still confining and thus provide the two point functions with the correct analytic structure. After a further integration of the heavy 'constituent' quarks one is left with an effective lagrangean written in terms of purely physical degrees of freedom. In this lagrangean the net effect of the residual gluon interactions is to contribute to the actual value of the coefficients. In fact, they do so by an amount that, while important, should not be the dominant one.

As in the previous paper we shall restrict ourselves to the case $N_F = 2$. We add also external vector,$\bar{V}_\mu(x)$, axial-vector, $\bar{A}_\mu(x)$, scalar, $\bar{S}(x)$ and pseudoscalar, $\bar{P}(x)$ sources in order to compute the correlators of corresponding quark currents and to analyze meson properties. These external fields are induced by a minimal coupling in the QCD Dirac operator (in euclidean notations)

$$\hat{D} \equiv i\gamma_\mu \left(\partial_\mu + \bar{V}_\mu + \gamma_5 \bar{A}_\mu\right) + i\left(\bar{S} + i\gamma_5 \bar{P}\right), \tag{1.1}$$

where $\langle S \rangle = m_q$, the diagonal matrix of current quark masses.

The low-energy effective lagrangean $\mathcal{L}_{ECQM}$ must be invariant under simultaneous left and right SU(2) rotations, $\Omega_R(x)$, $\Omega_L(x)$, of quark, chiral and external fields [2, 3]

$$\begin{aligned}
U &\to \Omega_R U \Omega_L^+ , \\
q_L \equiv \frac{1}{2}(1+\gamma_5) q &\to \Omega_L q_L , \\
q_R \equiv \frac{1}{2}(1-\gamma_5) q &\to \Omega_R q_R , \\
\bar{L}_\mu \equiv \bar{V}_\mu + \bar{A}_\mu &\to \Omega_L \bar{L}_\mu \Omega_L^+ + \Omega_L \partial_\mu \Omega_L^+ , \\
\bar{R}_\mu \equiv \bar{V}_\mu - \bar{A}_\mu &\to \Omega_R \bar{R}_\mu \Omega_R^+ + \Omega_R \partial_\mu \Omega_R^+ , \\
\bar{\mathcal{M}} \equiv \bar{S} + i\bar{P} &\to \Omega_R \bar{\mathcal{M}} \Omega_L^+ .
\end{aligned} \tag{1.2}$$

It is convenient to introduce the 'rotated','dressed' or 'constituent' quark fields [2]

$$Q_L \equiv \xi q_L , \qquad Q_R \equiv \xi^\dagger q_R , \qquad \xi^2 \equiv U , \tag{1.3}$$

which transform nonlinearly under $\mathrm{SU}_L(2) \otimes \mathrm{SU}_R(2)$ but identically for left and right quark components

$$\xi \longrightarrow h_\xi \xi \Omega_L^+ = \Omega_R \xi h_\xi^+ , \qquad Q_{L,R} \to h_\xi Q_{L,R}. \tag{1.4}$$

Changing to the 'dressed' basis implies the following replacements in the external vector, axial, scalar and pseudoscalar sources

$$\begin{aligned}
&\bar V_\mu \to v_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi - \partial_\mu\xi\xi^\dagger + \xi^\dagger\bar V_\mu\xi + \xi\bar V_\mu\xi^\dagger - \xi^\dagger\bar A_\mu\xi + \xi\bar A_\mu\xi^\dagger\right),\\
&\bar A_\mu \to a_\mu = \frac{1}{2}\left(-\xi^\dagger\partial_\mu\xi - \partial_\mu\xi\xi^\dagger - \xi^\dagger\bar V_\mu\xi + \xi\bar V_\mu\xi^\dagger + \xi^\dagger\bar A_\mu\xi + \xi\bar A_\mu\xi^\dagger\right),\\
&\bar{\mathcal M} \to \mathcal M = \xi^\dagger\bar{\mathcal M}\xi^\dagger\,.
\end{aligned} \tag{1.5}$$

Under $\mathrm{SU}_L(2)\otimes\mathrm{SU}_R(2)$ transformations

$$v_\mu \to h_\xi v_\mu h_\xi^+ + h_\xi\partial_\mu h_\xi^+\,, \qquad a_\mu \to h_\xi a_\mu h_\xi^+\,, \qquad \mathcal M \to h_\xi\mathcal M h_\xi^+\,. \tag{1.6}$$

In these variables the relevant part of ECQM action consists of three parts

$$\mathcal L_{ECQM} = \mathcal L_{ch} + \mathcal L_{\mathcal M} + \mathcal L_{vec}\,, \tag{1.7}$$

where $\mathcal L_{ch}$ accumulates the interaction of chiral fields and quarks in the chiral limit in the presence of vector and axial-vector external fields, $\mathcal L_{\mathcal M}$ extends the description for external scalar and pseudoscalar fields and, in particular, for massive quarks and, finally, $\mathcal L_{vec}$ contains operators generating meson states in vector and axial-vector channels.

More specifically

$$\begin{aligned}
\mathcal L_{ch} = {} & \mathcal L_0 + i\bar Q\left(\not{D} + M_0\right)Q + i\frac{4\delta f_0}{\Lambda^2}\bar Q a_\mu a_\mu Q + \\
& + \frac{G_{S0}}{4N_c\Lambda^2}\left(\bar Q_L Q_R + \bar Q_R Q_L\right)^2 - \frac{G_{P1}}{4N_c\Lambda^2}\left(-\bar Q_L\vec\tau Q_R + \bar Q_R\vec\tau Q_L\right)^2 + \\
& + \frac{G_{S1}}{4N_c\Lambda^2}\left(\bar Q_L\vec\tau Q_R + \bar Q_R\vec\tau Q_L\right)^2 - \frac{G_{P0}}{4N_c\Lambda^2}\left(-\bar Q_L Q_R + \bar Q_R Q_L\right)^2,
\end{aligned} \tag{1.8}$$

where

$$\begin{aligned}
Q &\equiv Q_L + Q_R\,,\\
\not{D} &\equiv \not\partial + \not{v} - \gamma_5\tilde g_A\,\not{a}\,.
\end{aligned} \tag{1.9}$$

with the axial coupling $\tilde g_A \equiv 1 - \delta g_A$ in the notations of [1]. The effective coefficients appearing in the above expression contain contributions from modes above $\Lambda_{CSB}$. The term $\mathcal L_0$ contains operators involving only chiral fields or external vector and axial-vector sources, such as for instance a term of the form

$$-\frac{f_0^2}{4}\,\mathrm{tr}\,a_\mu a_\mu\,. \tag{1.10}$$

We call this and similar pieces 'bare' contributions to the chiral effective lagrangean because they will be renormalized after integration of the low modes of the quark (and gluon) fields. Their coupling constants at the scale $\Lambda_{CSB}$ are expressible in terms of expectation values of the (high frequency) gluon field operators. The normalizing constant $\Lambda$ is taken to be $\Lambda_{CSB}$.

As the global chiral symmetry holds under simultaneous transformations of current quark fields, chiral fields and external sources, the chiral invariance of the different operators of the lagrangean is certainly provided in terms of constituent fields. Notice that the couplings of the four-fermion operators $G_{S0}, G_{S1}, G_{P0}$ and $G_{P1}$ are in general different. However in the large $N_c$ limit one expects that the identities $G_{S0} = G_{S1}$ and $G_{P0} = G_{P1}$ must hold. But the latter is severely broken due to the anomaly. Furthermore, the use of the equations of motion gives unequal contributions to all these constants, so we regard them as independent parameters in the effective lagrangean.

The massive part $\mathcal{L}_{\mathcal{M}}$ contains the following operators to the lowest relevant order

$$\begin{aligned}\mathcal{L}_{\mathcal{M}} = i\left(\frac{1}{2}+\epsilon\right)&\left(\bar{Q}_R\mathcal{M}Q_L + \bar{Q}_L\mathcal{M}^\dagger Q_R\right) + \\ +i\left(\frac{1}{2}-\epsilon\right)&\left(\bar{Q}_R\mathcal{M}^\dagger Q_L + \bar{Q}_L\mathcal{M}Q_R\right) + \\ +\operatorname{tr}&\left(c_0\left(\mathcal{M}+\mathcal{M}^\dagger\right) + c_5(\mathcal{M}+\mathcal{M}^\dagger)a_\mu a_\mu + c_8\left(\mathcal{M}^2 + \left(\mathcal{M}^\dagger\right)^2\right)\right), \qquad (1.11)\end{aligned}$$

where $\epsilon$ and $c_0, c_5, c_8$ are real coupling constants. The couplings $c_0, c_5, c_8$ (which only depend on chiral fields and external sources) are yet another instance of what we have called 'bare' terms. The reader will notice that we have changed slightly our notation with respect to our previous work in [1]. First the coefficients $c_i$ are labeled in a way that remind us of the coefficients of the effective chiral lagrangean to which they eventually contribute. Second, the matrix $\mathcal{M}$ introduced here is actually $\xi^\dagger\mathcal{M}\xi^\dagger$ in the notation of [1]. The present notation considerably simplifies our formulae.

Finally, the quark self-interactions in the vector and axial-vector channels, $\mathcal{L}_{vec}$, are implemented by

$$\begin{aligned}\mathcal{L}_{vec} = &-\frac{G_{V1}}{4N_c\Lambda^2}\bar{Q}\vec{\tau}\gamma_\mu Q\bar{Q}\vec{\tau}\gamma_\mu Q - \frac{G_{A1}}{4N_c\Lambda^2}\bar{Q}\vec{\tau}\gamma_5\gamma_\mu Q\bar{Q}\vec{\tau}\gamma_5\gamma_\mu Q - \\ &-\frac{G_{V0}}{4N_c\Lambda^2}\bar{Q}\gamma_\mu Q\bar{Q}\gamma_\mu Q - \frac{G_{A0}}{4N_c\Lambda^2}\bar{Q}\gamma_5\gamma_\mu Q\bar{Q}\gamma_5\gamma_\mu Q + \\ &+c_{10}\operatorname{tr}\left(U\bar{L}_{\mu\nu}U^\dagger\bar{R}_{\mu\nu}\right) + h_1\operatorname{tr}\left((\bar{L}_{\mu\nu})^2 + (\bar{R}_{\mu\nu})^2\right) + h_2\operatorname{tr}\left(\bar{S}^2+\bar{P}^2\right), \qquad (1.12)\end{aligned}$$

where the notations $\bar{L}_{\mu\nu}, \bar{R}_{\mu\nu}$ stand for the strengths of fields $L_\mu, R_\mu$ respectively and, again, the couplings $G_{V0}, G_{V1}, G_{A0}$ and $G_{A1}$ are, in general, different. (Of course in the large $N_c$ limit the relations $G_{V0} = G_{V1}$ and $G_{A0} = G_{A1}$ hold.) The term proportional to $c_{10}$ is a 'bare' term. The terms accompanying $h_1$ and $h_2$ do not couple to meson fields and, being manifestly chirally invariant, do not contribute to the chiral symmetry restoration sum rules which will be discussed below. We will omit further references to them.

Altogether we have eight $d = 6$ four-fermion operators. Of those we shall only retain those that will be required in our subsequent analysis and those where a fair comparison with phenomenology is possible in the SU(2) case. In practice this means that we retain the operators corresponding to the couplings $G_{S0}, G_{P1}, G_{V1}, G_{A1}$ which describe the phenomenology of $I = 1$ pseudoscalar, vector and axial-vector mesons. The iso-singlet scalar channel is essential in our analysis.

We complete this section with a remark on the correspondence between our notations and those ones in a manifestly chiral-symmetric extended [10, 25, 26] (see also [27]) NJL model with universal scalar, $G_S$, and vector, $G_V$ coupling constants:

$$G_{S0} = G_{P1} = 4\pi^2 G_S\,; \qquad G_{V1} = G_{A1} = 8\pi^2 G_V\,. \tag{1.13}$$

A general introduction to chiral effective lagrangean techniques can be found in [14, 15, 16]. Their extension to incorporate vector and axial-vector states is discussed in [17, 18, 19, 20]. The derivation of the chiral effective lagrangean via direct bosonization methods is discussed in [3, 21, 22, 23].

## 2. Bosonization of the ECQM with auxiliary fields

Let us add auxiliary fields in the scalar and pseudoscalar channels, $\widetilde{\Sigma}, \widetilde{\Pi} \equiv \widetilde{\Pi}^a\tau^a$, and in the vector and axial-vector channel, $\widetilde{W}_\mu^{(\pm)} = \widetilde{W}_\mu^{(\pm)a}\tau^a$, in order to bilinearize the lagrangean (1.7), (1.8) and (1.12) in fermion variables. We replace the four-fermion interaction by

$$\begin{aligned}\bar{Q}\left[i\widetilde{\Sigma} - \gamma_5\widetilde{\Pi} + \frac{1}{2}\gamma_\mu \widetilde{W}_\mu^{(+)} + \frac{1}{2}\gamma_\mu\gamma_5 \widetilde{W}_\mu^{(-)}\right]Q\, + \\ + N_c\Lambda^2\left[\frac{\widetilde{\Sigma}^2}{G_{S0}} + \frac{(\widetilde{\Pi}^a)^2}{G_{P1}} + \frac{\left(\widetilde{W}_\mu^{(+)a}\right)^2}{4G_{V1}} + \frac{\left(\widetilde{W}_\mu^{(-)a}\right)^2}{4G_{A1}}\right]\end{aligned} \tag{2.1}$$

and include an integration over the real auxiliary variables $\widetilde{\Sigma}, \widetilde{\Pi}^a, \widetilde{W}_\mu^{(+)a}, W_\mu^{(-)a}$. After integration of the fermionic degrees of freedom the auxiliary fields will, generally speaking, propagate. However, some redefinitions will be required. This is the reason for the tildes in (2.1).

The scalar block of fields in the Dirac operator (1.1) reads

$$\Sigma = M_0 + \widetilde{\Sigma} + \frac{1}{2}\left(\mathcal{M} + \mathcal{M}^\dagger\right) + \frac{4\delta f_0}{\Lambda^2} a_\mu a_\mu\,. \tag{2.2}$$

Likewise, we group all pseudoscalar fields in the Dirac action into the pseudoscalar block

$$\Pi = \widetilde{\Pi} + i\epsilon\left(\mathcal{M}^\dagger - \mathcal{M}\right). \tag{2.3}$$

Finally, the blocks of (anti-hermitean) vector and axial vector fields have the following form

$$V_\mu = v_\mu - \frac{1}{2} i \widetilde{W}_\mu^{(+)} , \qquad A_\mu = \tilde{g}_A a_\mu - \frac{1}{2} i \widetilde{W}_\mu^{(-)} . \tag{2.4}$$

The bosonization is completed by integration over the quark fields $\bar{Q}, Q$, which induces the quark loop effective action $\mathcal{W}_{1-loop}$, in terms of the determinant of (1.1). This determinant must be regularized with the help of a chirally symmetric regulator [28] and normalized at a scale $\mu$. The regulator suppresses frequencies above the cut-off $\Lambda$, already introduced as an arbitrary scale normalizing the four-fermion operators, and which is identified with the scale of chiral symmetry breaking, $\Lambda = \Lambda_{CSB}$. This cut-off is thus physical and its removal should not be attempted. On the other hand, $\mu$ is the subtraction point and it is quite arbitrary. The $\mu$ dependence drops from observables, provided that we define the coefficients of the effective lagrangean $(M_0, G_{S0}, \ldots)$ appropriately (see [1]) by introducing running couplings. We may, as we did in [1], choose the normalization $\mu \simeq \Lambda$. The parameters of the effective lagrangean are then defined as those at the CSB scale, which simplifies the expressions noticeably. In [1] a step-function was chosen as regulator. Had we used another regulator, the result (except for the logarithmically enhanced terms) would have been different, but so would the 'bare' terms in our effective action, which contain information about higher frequencies. The result would indeed be scheme dependent. However, since in practice we cannot really compute these 'bare' terms we are limited to using the (universal) logarithmic terms. This approximation is justified inasmuch as these terms are numerically dominant.

In particular, the effective potential for a constant $\Sigma$ field, $\langle \Sigma \rangle \equiv \Sigma_0$, can then easily be derived

$$V_{eff}(\Sigma_0) = N_c \left\{ \frac{\Lambda^2}{G_{S0}} (\Sigma_0 - M_0 - m_q)^2 + \frac{1}{8\pi^2} \Sigma_0^4 \left( \ln \frac{\Lambda^2}{\Sigma_0^2} + \frac{1}{2} \right) \right\} , \tag{2.5}$$

revealing a non-trivial minimum given by a solution of the mass-gap equation

$$\frac{\Lambda^2}{G_{S0}} (\Sigma_0 - M_0 - m_q) = -\frac{\Sigma_0^3}{4\pi^2} \ln \frac{\Lambda^2}{\Sigma_0^2} . \tag{2.6}$$

In the weak coupling regime the solution becomes $\Sigma_0 \simeq M_0 + m_q$ with $M_0 \gg m_q$ for light $u, d$ quarks. There are no corrections to this result in the large $N_c$ limit.

It follows from eq. (2.6) that it is natural to work not with the original parameters $G_{S0}$, $G_{A1}$, etc, but rather with

$$\begin{aligned} \bar{G}_S &= G_{S0} I_0 \frac{\Sigma_0^2}{\Lambda^2} , & \bar{G}_P &= G_{P1} I_0 \frac{\Sigma_0^2}{\Lambda^2} , & & \\ \bar{G}_V &= 2 G_{V1} I_0 \frac{\Sigma_0^2}{\Lambda^2} , & \bar{G}_A &= 2 G_{A1} I_0 \frac{\Sigma_0^2}{\Lambda^2} , & I_0 &\equiv \frac{1}{4\pi^2} \ln \frac{\Lambda^2}{\Sigma_0^2} . \end{aligned} \tag{2.7}$$

The parameters defined with bars are the ones controlling the departure from the 'natural' solution $\Sigma_0 = M_0$. The weak coupling regime corresponds to $\bar{G} \ll 1$. In [1]



JHEP10(1999)022

we defined similar couplings but containing $M_0$ rather than $\Sigma_0$. We have found that our expressions simplify when we use (2.7).

In this notation, and neglecting higher powers of $m_q$, the solution of the mass-gap eq. (2.6) reads

$$\Sigma_0(m_q) \simeq \Sigma_0(0) + m_q \frac{1}{1+3\bar{G}_S}\,; \qquad \Sigma_0(0) \equiv \Sigma_0 = \frac{M_0}{1+\bar{G}_S}\,. \tag{2.8}$$

We organize the effective action as a derivative expansion in inverse powers of $\Sigma_0$. We can thus write the full quark-loop effective action. Retaining the logarithmically enhanced part we get

$$\begin{aligned}\mathcal{L}_{1-loop} \simeq \frac{N_c}{16\pi^2}\ln\frac{\Lambda^2}{\Sigma_0^2}\,\mathrm{tr}\Bigg( & (\Sigma^2+\Pi^2)^2 + (\partial_\mu,\Sigma)^2 + [D_\mu^V,\Pi]^2 - \\ & - 4(A_\mu)^2\Sigma^2 - \{A_\mu,\Pi\}^2 - 4i[D_\mu^V,\Pi]A_\mu\Sigma + \\ & + 2i\partial_\mu\Sigma\{A_\mu,\Pi\} - \frac{1}{6}\left((F_{\mu\nu}^L)^2 + (F_{\mu\nu}^R)^2\right)\Bigg)\,,\end{aligned} \tag{2.9}$$

in terms of the fields defined in (2.2), (2.3), and (2.4). This lagrangean accumulates the one-loop quark effects and together with bare kinetic term for chiral fields in (1.8) and last lines of (1.11) and (2.1) forms the effective meson lagrangean in the presence of external fields.

As we will see later, $\Sigma_0 \simeq 200\,\mathrm{MeV}$ and it would seem that this expansion has an unacceptably small radius of convergence, certainly far below the scale $\Lambda_{CSB}$ up to which the effective theory is supposed to be valid. There are several reasons making the expansion a lot more useful than it would naively appear to be. The term of $\mathcal{O}(n)$ in the expansion has a numerical factor $\sim n^{-3}$; furthermore most of the terms with two and four derivatives (and certainly all those we retain) are further enhanced with a logarithmic factor. All things considered, the expansion is asymptotic at best. Physical arguments tell us which is the natural scale where the expansion stops making sense. It should be provided by the lowest mass not included in our low energy spectrum, roughly of order $\Lambda_{CSB}$. Non-logarithmically enhanced terms of higher order get contributions from higher dimensional operators too, so the consideration of terms with more derivatives is a futile exercise at this point: a lot of new unknown parameters would enter the game. But these terms, were we able to systematically take them into account, would enlarge the applicability of the model to non-logarithmically enhanced terms all the way up to $\Lambda_{CSB}$.

## 3. Constant $g_A$, masses and coupling constants of vector fields

Let us examine the effective lagrangean obtained after the integration of the quark fields in what concerns the axial-vector fields. There is a mass term

$$\Delta\mathcal{L} = \frac{N_c I_0 \Sigma_0^2}{4}\,\mathrm{tr}\left(\frac{1}{\bar{G}_A}\left(\widetilde{W}_\mu^{(-)}\right)^2 + \left(i2\tilde{g}_A a_\mu + \widetilde{W}_\mu^{(-)}\right)^2\right). \tag{3.1}$$

This term can be diagonalized [10, 25] by defining

$$i2\tilde{g}_A a_\mu + \widetilde{W}^{(-)}_\mu = i2 g_A a_\mu + \frac{1}{\lambda_-} W^{(-)}_\mu \,, \tag{3.2}$$

with

$$g_A = \frac{\tilde{g}_A}{1+\bar{G}_A} \,, \tag{3.3}$$

which differs from the related expression in the extended NJL model [10, 25] due to presence of a bare constant $\tilde{g}_A$.

The constant $\lambda_-$ and its vector counterpart $\lambda_+$ are determined by requiring the proper normalization of the kinetic term for physical vector fields $W^{(\pm)}_\mu$. The appropriate normalization constants coincide in (2.9)

$$\lambda_+^2 = \lambda_-^2 = \frac{N_c I_0}{6} \,. \tag{3.4}$$

The masses of vector mesons can be evaluated in the large-log approximation from (2.1) and (2.9).

$$m_V^2 = \frac{6\Sigma_0^2}{\bar{G}_V} \,, \tag{3.5}$$

whereas the axial-meson mass is derived from (3.1)

$$m_A^2 = 6\Sigma_0^2 \frac{1+\bar{G}_A}{\bar{G}_A} = 6\Sigma_0^2 \frac{\tilde{g}_A}{\tilde{g}_A - g_A} \,. \tag{3.6}$$

Therefore

$$g_A = \tilde{g}_A \frac{m_A^2 - 6\Sigma_0^2}{m_A^2} \,. \tag{3.7}$$

Among other characteristics of vector mesons, the co upling constants to external vector fields are of main importance. They are defined by the following term in the lagrangean

$$\Delta\mathcal{L} = \frac{i}{4} \,\mathrm{tr} \left( f_V W^{(+)}_{\mu\nu} \left( \xi \bar{L}_{\mu\nu} \xi^\dagger + \xi^\dagger \bar{R}_{\mu\nu} \xi \right) + f_A W^{(-)}_{\mu\nu} \left( \xi \bar{L}_{\mu\nu} \xi^\dagger - \xi^\dagger \bar{R}_{\mu\nu} \xi \right) \right) \,, \tag{3.8}$$

where $W^{(+)}_{\mu\nu}$ and $W^{(-)}_{\mu\nu}$ are the field strength tensors constructed with $W^{(+)}_\mu$ and $W^{(-)}_\mu$, respectively. From the previous expression one easily obtains that

$$f_V = \lambda_+ \,; \qquad f_A = g_A \lambda_- = g_A f_V \,. \tag{3.9}$$

## 4. Scalar and pseudoscalar sector: mass spectrum and decay constants

We begin by determining the pion decay constant which can be found by adding the bare pion kinetic term, (1.8), and the one obtained from the quark loop, shown in (3.1), after shifting the fields (3.2)

$$F_0^2 = f_0^2 + N_c \Sigma_0^2 I_0 g_A \tilde{g}_A \,. \tag{4.1}$$

Recalling that $a_\mu \simeq 2\bar{A}_\mu - i\partial_\mu \pi/F_0$ the bare kinetic term for pseudoscalar fields is given by

$$\Delta\mathcal{L} = \frac{1}{4}\,\mathrm{tr}\left((\partial_\mu\widetilde{\pi})^2 + \frac{N_c I_0 g_A}{\widetilde{g}_A}(\partial_\mu\widetilde{\Pi})^2 - \frac{2N_c I_0 \Sigma_0 g_A}{F_0}\partial_\mu\widetilde{\Pi}\partial_\mu\widetilde{\pi}\right) + \\ + \frac{1}{4}\,\mathrm{tr}\left(m_\pi^2\widetilde{\pi}^2 + \frac{4N_c I_0 \Sigma_0^2 \epsilon m_\pi^2}{\bar{G}_P B_0 F_0}\widetilde{\pi}\widetilde{\Pi}\right), \tag{4.2}$$

where we have performed a further redefinition of the $W_\mu^{(-)}$ field so as to cancel the $W_\mu^{(-)}\partial^\mu\widetilde{\Pi}$ mixing, which brings about another contribution to the $\widetilde{\Pi}$ kinetic term.

The pion mass is generated by the quark condensate

$$C_q = i\langle\bar{q}q\rangle_{eucl} = \left(2c_0 + \frac{N_c}{4\pi^2}\Sigma_0^3 \ln\frac{\Lambda^2}{\Sigma_0^2}\right) \equiv -B_0 F_0^2\,, \tag{4.3}$$

according to the Gell-Mann-Oakes-Renner formula, $m_\pi^2 = 2m_q B_0$ and the masses of the $u, d$ quarks are taken equal for simplicity. The constant $c_0$ (which was named $c_1$ in [1]) is required to have a renormalization-group invariant effective potential, as explained in [1].

In the chiral limit, neglecting with the pion mass, one diagonalizes the kinetic term with the help of the following transformations

$$\widetilde{\pi} = \widetilde{\pi}' + \frac{\sqrt{1-\delta^2}}{\delta}\widetilde{\Pi}'\,; \qquad \widetilde{\Pi} = \frac{1}{\delta}\sqrt{\frac{\widetilde{g}_A}{g_A N_c I_0}}\,\widetilde{\Pi}'\,, \tag{4.4}$$

where we have used the following notation: $\delta \equiv f_0/F_0$. As a result the heavy pseudoscalar mass is

$$m_\Pi^2 = \frac{2\Sigma_0^2\widetilde{g}_A}{\delta^2 g_A}\left(\frac{1}{\bar{G}_P} + 1\right). \tag{4.5}$$

In the massless limit one can check that heavy pseudoscalar mesons completely decouple from external axial fields and only the pion couples to them through the vertex $\sim \bar{A}_\mu\partial_\mu\widetilde{\pi}'$. For massive pions one finds in (4.2) an additional mixing between the fields $\widetilde{\pi}$ and $\widetilde{\Pi}$. After a further diagonalization of the mass term (for light pions, to the first order in $m_\pi^2$),

$$\widetilde{\pi}' \simeq \pi + d_1\frac{m_\pi^2}{m_\Pi^2}\Pi\,; \qquad \widetilde{\Pi}' \simeq -d_1\frac{m_\pi^2}{m_\Pi^2}\pi + \Pi\,; \\ d_1 = \frac{\sqrt{1-\delta^2}}{\delta}\left(\frac{2\Sigma_0\epsilon}{\bar{G}_P g_A B_0} + 1\right), \tag{4.6}$$

and the weak decay coupling constant for heavy $\Pi$ meson is found to be

$$F_\Pi = F_0 d_1 \frac{m_\pi^2}{m_\Pi^2(0)}\,. \tag{4.7}$$

The scalar mass is obtained directly from the lagrangean (2.1) and (2.9) deriving the quadratic form of the fluctuation, $\Sigma = \Sigma_0 + \tilde{\sigma}$

$$m_\sigma^2 = 2\Sigma_0^2\left(\frac{1}{\bar{G}_S} + 3\right) . \tag{4.8}$$

The physical scalar field is given by $\sigma = \tilde{\sigma}\sqrt{N_c I_0}$. The reader can easily verify that all the above formulae reduce to the ones of [1] after taking $\tilde{g}_A = g_A$, i.e. $\bar{G}_A = \infty$.

## 5. Chiral symmetry restoration and Weinberg sum rules

It was observed in [1] that a useful way of constraining the coefficients of the ECQM was provided by requiring that at $\mu = \Lambda_{CSB}$ there is an exact matching between the effective theory and QCD (including both perturbative and non-perturbative contributions) in those channels which are sensitive to chiral symmetry breaking. The only example which was explicitly worked out in our previous work was the difference between scalar and pseudoscalar Green functions. In QCD this difference behaves as $1/p^4$, $p^2$ being the squared momentum.

The scale of the matching is, in principle, arbitrary. For it to be useful, it should, however, fulfill the following requirements. First it should be larger than the masses of the states included in the effective lagrangean and lower than the masses of the omitted resonances. Furthermore it must be in a region where perturbative QCD is still applicable to determine the coefficients of the OPE. The CSB scale at around 1.3 GeV meets all these requirements when the lowest lying massive resonances are included in all channels.

Let us then exploit the constraints based on matching the chiral symmetry restoration properties of QCD and the effective theory at high energies. For this purpose we focus on two-point correlators of color-singlet quark currents in euclidean space-time

$$\begin{aligned}
&\Pi_C(p^2) = \int d^4x\ \exp(ipx)\ \langle T\left(\bar{q}\Gamma q(x)\,\bar{q}\Gamma q(0)\right)\rangle, \\
&C \equiv S, P, V, A\,; \qquad \Gamma = i,\ \gamma_5\tau^a,\ \gamma_\mu\tau^a,\ \gamma_\mu\gamma_5\tau^a\,.
\end{aligned} \tag{5.1}$$

In the chiral limit the scalar correlator $\Pi_S$ and the pseudoscalar one $\Pi_P^{aa}$ coincide at all orders in perturbation theory and also at leading order in the non-perturbative O.P.E. [29, 30, 31] (see also [1, 32, 33]). In fact

$$\begin{aligned}
&\left(\Pi_P^{aa}(p^2) - \Pi_S(p^2)\right)_{p^2\to\infty} \equiv \frac{\Delta_{SP}}{p^4} + \mathcal{O}\left(\frac{1}{p^6}\right), \\
&\Delta_{SP} \simeq 24\pi\alpha_s C_q^2 \simeq 24 B_0^2 F_0^4\,,
\end{aligned} \tag{5.2}$$

where the vacuum dominance hypothesis [29] has been applied in the large-$N_c$ limit and the round value $\alpha_s(1.2\,\mathrm{GeV}) \simeq 1/3$ is taken for simplicity [29]. Therefore the difference (5.2) represents a genuine order parameter of CSB in QCD.

The same is true for the difference between the vector, $\Pi_V^{aa}$ and axial-vector, $\Pi_A^{aa}$ correlators [29, 35, 36, 37]

$$\left(\Pi_V^{aa}(p^2) - \Pi_A^{aa}(p^2)\right)_{p^2\to\infty} \equiv \frac{\Delta_{VA}}{p^6} + \mathcal{O}\left(\frac{1}{p^8}\right),$$
$$\Delta_{VA} = -16\pi\alpha_s C_q^2 \simeq -16 B_0^2 F_0^4\,, \tag{5.3}$$

where we have defined in the $V, A$ channels

$$\Pi_{\mu\nu}^{aa} \equiv \left(-\delta_{\mu\nu}p^2 + p_\mu p_\nu\right)\Pi^{aa}(p^2)\,. \tag{5.4}$$

On the other hand, in the large-$N_c$ limit all correlators are saturated by narrow resonances [40, 41]

$$\Pi_P^{aa}(p^2) - \Pi_S(p^2) = \sum_n \left[\frac{Z_n^P}{p^2+m_{P,n}^2} - \frac{Z_n^S}{p^2+m_{S,n}^2}\right],$$
$$\Pi_V^{aa}(p^2) - \Pi_A^{aa}(p^2) = \sum_n \left[\frac{Z_n^V}{p^2+m_{V,n}^2} - \frac{Z_n^A}{p^2+m_{A,n}^2}\right] - \frac{4F_0^2}{p^2}\,. \tag{5.5}$$

As the two above differences decrease rapidly as $p^2$ increases, one can rightly expect that only the lowest lying resonances will contribute to (and hence will be sensitive to) the constraints from CSB restoration.

Thus the resonances described by the ECQM; namely a scalar particle, two pseudoscalar, a vector and an axial-vector ones, should provide the leading asymptotic terms in (5.2) and (5.3). It implies two sets of sum rules which in the vector channel are known as the Weinberg Sum Rules [42]. In particular, in the scalar channel one obtains

$$c_8 + \frac{N_c\Sigma_0^2 I_0}{8\bar{G}_S} - \frac{4\epsilon^2 N_c \Sigma_0^2 I_0}{8\bar{G}_P} = 0\,, \tag{5.6}$$
$$Z_\sigma = Z_\pi + Z_\Pi, \qquad Z_\pi = 4B_0^2F_0^2\,, \tag{5.7}$$
$$Z_\sigma m_\sigma^2 = Z_\Pi m_\Pi^2 + \Delta_{SP}\,. \tag{5.8}$$

The first relation determines the bare chiral constant $c_8$ so that to compensate completely the local contributions from four-fermion interaction. Two other ones reduce the number of independent ECQM coupling constants.

The $\sigma$ and $\Pi$ masses have already been derived in the previous sections. To impose the chiral symmetry restoration constraints we need to know the appropriate residues. These can be obtained from the part of the ECQM lagrangean which couples scalar and pseudoscalar fields to their sources. Namely

$$\Delta\mathcal{L} = \left[-\frac{2N_c\Sigma_0^2 I_0}{\bar{G}_S}\tilde{\sigma}\bar{S} - \frac{4N_c\Sigma_0^2 I_0\epsilon}{\bar{G}_P}\tilde{\Pi}^a\bar{P}^a - 2B_0F_0^2\tilde{\pi}^a\bar{P}^a\right]$$
$$= -\alpha\sigma\bar{S} - \alpha\beta\pi^a\bar{P}^a - \frac{\alpha\left(2\tilde{\epsilon}\gamma + \beta\sqrt{1-\delta^2}\right)}{\delta}\Pi^a\bar{P}^a\,, \tag{5.9}$$

where the last line is written in terms of the physical fields, i.e. after the diagonalization (4.4). We have used the following notation

$$\alpha \equiv \frac{2\Sigma_0^2\sqrt{N_c I_0}}{\bar{G}_S}\,, \qquad \beta \equiv \frac{B_0 F_0 \bar{G}_S}{\sqrt{N_c I_0}\Sigma_0^2}\,, \qquad \gamma \equiv \frac{\bar{G}_S}{\bar{G}_P}\,, \qquad \tilde{\epsilon} = \sqrt{\frac{\tilde{g}_A}{g_A}}\,\epsilon\,. \tag{5.10}$$

Comparing the above expressions to the equivalent ones without vector channels, the only modification consists in the replacement $\epsilon \to \tilde{\epsilon}$. Therefore all other relations hold as well. Namely the sum rules (5.7) and (5.8) lead to

$$\beta = \frac{\sqrt{1-\frac{m_\sigma^2}{m_\Pi^2}}}{\sqrt{1-\frac{\Delta_{SP}}{Z_\pi m_\Pi^2}}} \simeq \sqrt{1-\frac{m_\sigma^2}{m_\Pi^2}}\,. \tag{5.11}$$

The approximation made in the previous expression is justified since

$$\frac{\Delta_{SP}}{Z_\pi m_\Pi^2} \simeq \frac{6F_0^2}{m_\Pi^2} \simeq 0.03\,. \tag{5.12}$$

Thus, from phenomenology [34] $|\beta| < 1$. The relation (5.7) gives

$$2\tilde{\epsilon}\gamma = -\beta\sqrt{1-\delta^2} \pm \delta\sqrt{1-\beta^2}\,, \tag{5.13}$$

and, since $|\delta| < 1$, it follows that $|2\tilde{\epsilon}\gamma| \leq 1$.

When taking into account the sum rule (5.13) one can express the parameter $d_1$ in (4.6) solely as a function $\beta$

$$d_1 = \pm\frac{\sqrt{1-\beta^2}}{\beta}\,. \tag{5.14}$$

The scalar decay constant is defined through the relation

$$2B_0 F_\sigma = \sqrt{Z_\sigma}\,. \tag{5.15}$$

Therefore

$$F_\sigma = \frac{\sqrt{N_c I_0}\Sigma_0^2}{B_0 \bar{G}_S} = \frac{F_0}{\beta}\,. \tag{5.16}$$

It is illustrative to compare the realization of these sum rules in the ECQM to the one in the usual NJL model, which contains a (light) massive scalar and a massless pion. In the corresponding correlators one retains only those poles and therefore the terms accompanying the constant $Z_\Pi$ should be dropped. The first sum rule then gives $Z_\pi = Z_\sigma$, i.e. the characteristic relation of the linear $\sigma$-model. The second sum rule contains the quantity $Z_\pi m_\sigma^2$. In the NJL model

$$Z_\pi m_\sigma^2 = 16 C_q^2 \frac{\Sigma_0^2}{F_0^2} \simeq C_q^2 \frac{64\pi^2}{N_c g_A \ln\left(\Lambda^2/\Sigma_0^2\right)}\,. \tag{5.17}$$

To derive this result the well-known expression for $F_0$ in the NJL model has been used (equal to our eq. (4.1) with $f_0 = 0$, $\tilde{g}_A = 1$). Chiral symmetry restoration implies that (5.17) must be equal to

$$24\pi\alpha_s C_q^2 \simeq C_q^2 \frac{288\pi^2}{11N_c \ln(\Lambda^2/\Sigma_0^2)}\,, \tag{5.18}$$

which is obviously not the case since this would require $g_A \simeq 2.5$! This result is remarkably independent of colors, flavors and scales.

In the vector channel one derives a similar set of sum rules

$$c_{10} = 0\,, \tag{5.19}$$
$$f_V^2 m_V^2 = f_A^2 m_A^2 + F_0^2\,, \tag{5.20}$$
$$f_V^2 m_V^4 = f_A^2 m_A^4\,, \tag{5.21}$$
$$f_V^2 m_V^6 - f_A^2 m_A^6 \simeq -4\pi\alpha_s C_q^2 \simeq -4B_0^2 F_0^4\,, \tag{5.22}$$

Eqs. (5.20) and (5.21) are the celebrated Weinberg sum rules, while the last equation (5.22) was obtained in [35] (related considerations can be found also in [37, 38, 39]). We have taken into account that

$$Z_n^V = 4f_{V,n}^2 m_{V,n}^2\,, \qquad Z_n^A = 4f_{A,n}^2 m_{A,n}^2\,. \tag{5.23}$$

The first constraint fixes the bare chiral constant $c_{10}$ and implies the absence of bilinear local operators in the external fields. The sum rules (5.20) and (5.21) determine the constants $f_V, f_A$ in terms of vector meson masses and $F_0$. Since in section 3 these same constants have been determined in terms of $I_0$ and $g_A$ we conclude that

$$g_A = \frac{m_V^2}{m_A^2} = \frac{\bar{G}_A}{\bar{G}_V(1+\bar{G}_A)}\,, \qquad f_V^2 = \frac{F_0^2}{m_V^2(1-g_A)} = \frac{N_c I_0}{6}\,, \qquad f_A^2 = \frac{g_A^2 F_0^2}{m_V^2(1-g_A)}\,. \tag{5.24}$$

With these constants one can try to saturate the last sum rule (5.22) which would imply

$$F_0^2 m_V^2 m_A^2 \simeq 4B_0^2 F_0^4\,, \tag{5.25}$$

failing to be satisfied for realistic vector and axial-vector meson masses and for $B_0 \simeq 1.5\,\mathrm{GeV} \simeq 2m_V$ [14, 43] as $m_A \gg 4F_0$. At this point the lowest-resonance approximation is not anymore satisfactory which reminds us about the situation of the NJL model in the scalar channel. Certainly higher-mass, excited vector (and axial-vector) resonances are needed to correct the asymptotic behavior if (5.22) is to be fulfilled.



JHEP10(1999)022

## 6. Chiral constants, fit and discussion

We begin this section by obtaining the values of the chiral constants $L_5$, $L_8$ and $L_{10}$. The derivation of the first two constants exactly follows the procedure outlined in [1]. The results are

$$L_5 = \frac{N_c \Sigma_0 I_0 g_A^2}{8B_0(1+3\bar{G}_S)} \,. \tag{6.1}$$

$$L_8 = \frac{1}{64B_0^2}\left(\frac{Z_\sigma}{m_\sigma^2} - \frac{Z_\Pi}{m_\Pi^2}\right) = \frac{F_0^2}{16}\left(\frac{1}{m_\sigma^2} + \frac{1}{m_\Pi^2}\right)\left(1 - \frac{\Delta_{SP}}{Z_\pi(m_\sigma^2+m_\Pi^2)}\right), \tag{6.2}$$

The term proportional to $\Delta_{SP}$ is very small and, in practice, negligible

$$\frac{\Delta_{SP}}{Z_\pi(m_\sigma^2+m_\Pi^2)} \simeq \frac{6F_0^2}{m_\sigma^2+m_\Pi^2} < 0.03 \,. \tag{6.3}$$

Thus one expects that

$$L_8 > \frac{F_0^2}{8m_\Pi^2} \,. \tag{6.4}$$

Having considered the vector and axial-vector channels, we can also estimate the low energy limit of the difference vector and axial vector correlators and obtain in this way the value of the chiral constant $L_{10}$. In the chiral lagrangean, $L_{10}$ parameterizes the operator

$$-L_{10}\,\mathrm{tr}\left(U\bar{L}_{\mu\nu}U^\dagger\bar{R}_{\mu\nu}\right) \,. \tag{6.5}$$

As $c_{10}=0$, $L_{10}$ is saturated by vector and axial-vector exchange. From (3.8)

$$L_{10} = \frac{1}{4}\left(f_A^2 - f_V^2\right) = -\frac{F_0^2(1+g_A)}{4m_V^2} \,. \tag{6.6}$$

To proceed to an overall fit of the coefficients of the ECQM and in order to make some physical predictions, let us first specify the input parameters. As such we take $F_0 = 90\,\mathrm{MeV}$, $m_\pi^2 = 140\,\mathrm{MeV}$. Then [14, 43] $\hat{m}_q(1\,\mathrm{GeV}) \simeq 6\,\mathrm{MeV}$, $B_0(1\,\mathrm{GeV}) \simeq 1.5\,\mathrm{GeV}$, and, besides, the phenomenological value for the heavy pion mass $m_\Pi \simeq 1.3\,\mathrm{GeV}$ [34]. We also take the vector meson masses, $m_V = m_\rho = 770\,\mathrm{MeV}$ and $m_A = m_{a1} \simeq 1.1 \div 1.2$ MeV, as known parameters, though we will not be able to fit both of them so that to satisfy all sum rules (see discussion in previous section).

We start by determining the scalar meson mass based on estimates [14, 15, 16] for the chiral constant $L_8 = (0.9 \pm 0.4)\times 10^{-3}$

$$m_\sigma^2 = m_\Pi^2\left(\frac{16L_8 m_\Pi^2}{F_0^2} - 1\right)^{-1} \simeq (900 \pm 300)\,\mathrm{MeV} \,. \tag{6.7}$$

Employing the mean value of $m_\sigma$, we determine in turn $\beta \simeq 0.71$.

As discussed, it is not reliable to use (5.22), so we will omit any further reference to it. As for the second Weinberg sum rule (5.21), we will first assume that it holds (with only one resonance in each channel, that is) to find that the overall fit is only marginally consistent. Relaxation of the sum rule (5.21) will then allow for a much better fit.

We invoke eq. (5.24) to parameterize $I_0$ in terms of $g_A$

$$I_0 = \frac{6F_0^2}{N_c m_V^2 (1-g_A)}, \tag{6.8}$$

and eqs. (4.8) and (5.10) to find the dependence of $\bar{G}_S$ and $\Sigma_0$ on $g_A$

$$\bar{G}_S = \frac{\beta m_\sigma^2}{\sqrt{6}\, B_0 m_V \sqrt{1-g_A}} - \frac{1}{3}, \qquad \Sigma_0^2 = \frac{1}{6} m_\sigma^2 \left(1 - \frac{2B_0 m_V \sqrt{1-g_A}}{\beta m_\sigma^2 \sqrt{6}}\right). \tag{6.9}$$

With the help of these formulae one evaluates $L_5$

$$L_5 = \frac{F_0^2}{4 m_V m_\sigma \beta} \frac{g_A^2}{\sqrt{1-g_A}} \sqrt{1 - \frac{2B_0 m_V}{\beta m_\sigma^2 \sqrt{6}} \sqrt{1-g_A}}\,. \tag{6.10}$$

Consistency with real values of squared roots implies that

$$1 > g_A \geq \min \left[1 - \frac{3m_\sigma^4}{2B_0^2 m_V^2} \left(1 - \frac{m_\sigma^2}{m_\Pi^2}\right)\right] \simeq 0.48\,. \tag{6.11}$$

This is the absolute minimum of $g_A$, provided that $\Delta_{SP}$ can be safely neglected. This minimum value of $g_A$ is attained for $m_\sigma = 1060\,\mathrm{MeV}$. If we use the central value $m_\sigma \simeq 900\,\mathrm{MeV}$ one has $g_A \geq 0.60$ for $L_5$ to be real. However, the latter constant is bounded from the phenomenology of light pseudoscalar mesons to be $L_5 = (1.4 \pm 0.5) \times 10^{-3}$ for sufficiently heavy scalar mesons. Always sticking ourselves to the central value $m_\sigma \simeq 900\,\mathrm{MeV}$, the lower value for $L_5$, $L_5 \simeq 0.9 \times 10^{-3}$ is provided by $g_A \simeq 0.66$ and the mean value, $L_5 \simeq 1.4 \times 10^{-3}$ is provided by $g_A \simeq 0.71$.

On the other hand $g_A$ appears in the relation (5.24) between the masses of vector and axial vector mesons, $m_A^2 = m_V^2/g_A$ (that, again, assumes that the second Weinberg sum rule is saturated with only one resonance). Then for the allowed range of $L_5$ one gets $m_A \leq 0.95\,\mathrm{GeV}$. Hence, in order to be as close to the phenomenological value of $m_A = 1.1 \div 1.2\,\mathrm{GeV}$ as possible we adopt the lowest possible value for $g_A$ compatible with the experimentally allowed range for $L_5$ and therefore $g_A \simeq 0.66$ if $m_\sigma = 900\,\mathrm{MeV}$. For a heavier scalar meson with $m_\sigma = 1060\,\mathrm{MeV}$ one reaches the lowest $L_5$ for $g_A \simeq 0.62$ and predicting $m_A \simeq 1\,\mathrm{GeV}$.

A low value for $g_A$ triggers an unwanted effect, namely, the diminishing of the dynamical quark mass $\Sigma_0$ as it follows from (6.9). To optimize the fit one has to accept larger values for $g_A$ and/or heavier masses for scalar meson. In particular, for the mean values of $L_5$ and $m_\sigma$, i.e. for $g_A \simeq 0.71$, which leads nevertheless to unacceptably low values for $m_A$, one gets $\Sigma_0 \simeq 150\,\mathrm{MeV}$. The dynamical quark mass is maximal, $\Sigma_0 \simeq 190\,\mathrm{MeV}$, for $m_\sigma \sim 1.1\,\mathrm{GeV}$.

Thus we see that the extension of the ECQM to the vector and axial-vector channel adds very stringent constraints. While the overall fit is not bad, it is not brilliant either. In particular the tendency to underestimate the axial meson mass is evident and clearly means that, as already seen when discussing (5.22), with only one set of vector and axial vector states one cannot effectively saturate the Weinberg sum rule (5.21) either (see also the hints in [35]). Therefore let us accept that only the relation (5.20) holds with a reasonable accuracy and the next one does not. This implies a modification of eq. (5.24) in the following way

$$f_V^2 = \frac{F_0^2}{m_V^2(1-g_A^2\xi)} = \frac{N_c I_0}{6}, \qquad f_A^2 = \frac{g_A^2 F_0^2}{m_V^2(1-g_A^2\xi)}, \qquad \xi = \frac{m_A^2}{m_V^2} \neq \frac{1}{g_A}. \quad (6.12)$$

The term $\sqrt{1-g_A}$ in eqs. (6.9) and (6.10) should be replaced by $\sqrt{1-g_A^2\xi}$. The parameter $\xi$ takes the value $\xi \simeq 2.43$ for $m_A \simeq 1.2\,\mathrm{GeV}$.

Let us perform an optimal fit. For $m_\sigma \simeq 1\,\mathrm{GeV}$ one finds $\beta \simeq 0.64$ and $L_8 \simeq 0.8\times 10^{-3}$. For $g_A = 0.55$ one obtains $L_5 = 1.2\times 10^{-3}$ and $\Sigma_0 \simeq 200\,\mathrm{MeV}$. Then the vector and axial vector couplings are $f_V = 0.22$ and $f_A = 0.12$ to be compared with the experimental values [44] from the decay $\rho^0 \to e^+e^-$, $f_V \simeq 0.20\pm 0.01$, and from the decay $a_1 \to \pi\gamma$, $f_A = 0.10\pm 0.02$. Then from eqs. (3.5), (3.6) and (3.7) one derives that $\bar{G}_V \simeq 0.25$, $\bar{G}_A \simeq 0.2$, $\tilde{g}_A \simeq 0.66$. In addition $\bar{G}_S \simeq 0.11$ from eq. (4.8). With these values, $I_0 \simeq 0.1$ and $\Lambda \simeq 1.3\,\mathrm{GeV}$. Then the bare pion coupling takes the value $f_0 \simeq 62\,\mathrm{MeV}$ and for the rest of the parameters we find: $\delta \simeq 0.7$, $\bar{G}_P \simeq 0.13$, $\gamma \simeq 0.85$, and either $\epsilon \simeq 0.05$ or $\epsilon \simeq -0.51$. The naive QCD value $\epsilon \simeq 0.5$ is disfavored. Finally,

$$|d_1| = 1.2\,, \qquad F_\Pi = 0.8\times 10^{-2} F_\pi\,, \qquad F_\sigma = 1.6 F_\pi\,. \quad (6.13)$$

The constant $F_\Pi$ h as been estimated by different methods [19, 20, 45, 46, 47]) to correspond to $|d_1| = 1 \div 3$. But it is not yet experimentally measured.

## 7. Conclusions

In [1] we established that the QCD effective action that is appropriate below the scale of chiral symmetry breaking must contain chiral fields as well as quarks and gluons. The effective action must also include higher dimensional operators with relatively weak coupling constants and a relatively large constituent mass. In this paper we confirm that the optimal fit to meson physics, including vector and axial-vector channels, favors weakly coupled four-fermion operators, in contradistinction to the usual NJL model. The strength of the coupling is always referred to the constituent mass scale $\Sigma_0$.

Let us summarize once more the approximations used to derive the meson characteristics. The most crucial ones are the large $N_c$ and leading-log approximations.

The first one is equivalent to the neglect of meson loops. The second one, in fact, is fully compatible with quark confinement (due to the residual gluon interactions) as quark-antiquark threshold contributions are suppressed in two-point functions in the large log approximation. Furthermore logs are universal and independent of the method used to separate among low and high frequencies. The accuracy of this approximation also depends on the influence of heavy mass resonances which are not included into the particle content of the ECQM. One can actually improve of the leading-log approximation with the help of higher dimensional operators not retained in the ECQM lagrangean. But any extension of ECQM should be clearly accompanied with the opening of new meson channels with physical resonances of higher mass and spin.

In this paper we have used a more conventional [14, 15, 16] value for $L_5 = (1.4 \pm 0.5) \times 10^{-3}$ as compared to $L_5 = 2.2 \times 10^{-3}$ from [48] because the latter one was obtained in the assumption of a very light scalar meson. However our analysis has made evident that the low-mass scalar quarkonium bound state should be quite heavy ($m_\sigma \sim 1\,\mathrm{GeV}$). Yet it may not be the lightest state as another scalar meson supposedly exists as a $\pi\pi$ scattering state [49] with a mass of order 500–600 MeV.

The final estimates for effective coupling constants in the scalar and pseudoscalar channels are shown to be weakly dependent on the presence of vector mesons and roughly coincide to those ones in [1]. In particular, the dominance of two (or the second) mass terms in (11) is again proven.

The set of CSR constraints predicts unambiguously the value of axial quark coupling to pions quite a different from unity. It may be close to 0.5 conjectured in [36] but it seems to be unlikely that it reaches this value due to its being bounded from below because of $L_5$.

The predictivity of the ECQM is substantially provided by the CSR constraints, now enlarged with the Weinberg sum rules. However only the first of them is well compatible with the particle content of ECQM while the second and the third ones need at least one more heavy resonance.

For completeness we clarify in more detail the consistency of the full set of vector-channel sum rules (5.20)-(5.22) in the system with two vector and one axial-vector resonances. This case was firstly analyzed in [35]. A reasonable precision is provided by the fit

$$\begin{gathered} m_\rho = 770\,\mathrm{MeV}, \qquad m_{a1} = 1170\,\mathrm{MeV}, \qquad m_{\rho'} = 1420\,\mathrm{MeV}\,, \\ f_\rho = 0.18, \qquad f_{a1} = 0.11, \qquad f_{\rho'} = 0.05\,, \end{gathered} \tag{7.1}$$

to be compared to the experimental data [34] and some phenomenological estimations [44]

$$\begin{gathered} m_\rho = 770\,\mathrm{MeV}\,, \qquad m_{a1} = (1230 \pm 40)\,\mathrm{MeV}\,, \qquad m_{\rho'} = (1465 \pm 25)\,\mathrm{MeV}\,, \\ f_\rho = 0.20 \pm 0.01\,, \qquad f_{a1} = 0.10 \pm 0.02\,, \end{gathered} \tag{7.2}$$



JHEP10(1999)022

as to the value of $f_{\rho'}$ it has not been yet measured. We consider this precision to be excellent for the leading large-$N_c$ approximation (i.e. without any unitarity corrections) and from this we conclude that the second Weinberg sum rule and the last one (5.22) indeed should be applied only to the three-resonance system.

By adding two more parameters ($G_A$ and $G_V$) we have been able to predict a large number of physical parameters in addition to those already determined in [1], such as $f_A$, $f_V$, $m_A$, $m_V$, $L_{10}$ and, in particular, we get a very clear handle on $g_A$. Perhaps more importantly, the ECQM appears to be very robust and allows for this extension without any problems.

## Acknowledgments

This work is supported by EU Network EURODAPHNE, CICYT grant AEN98-0431 and CIRIT grant 1998SRG 00026. A.A. is supported by the Generalitat de Catalunya (Program PIV 1999) and partially by grants RFFI 98-02-18137 and GRACENAS 6-19-97. We are grateful to R. Tarrach for useful discussions and encouragement.



## References

[1] A.A. Andrianov, D. Espriu, R. Tarrach, *Nucl. Phys.* **B 533** (1998) 429 [hep-ph/9803232].

[2] H. Georgi, A. Manohar, *Nucl. Phys.* **B 234** (1984) 189; *Weak interactions and modern particle theory*, Addison-Wesley, Redwood City 1984.

[3] D. Espriu, E. de Rafael, J. Taron, *Nucl. Phys.* **B 345** (1990) 22.

[4] Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122** (1961) 345.

[5] U.-G. Meissner, *Phys. Rep.* **161** (1988) 213.

[6] H. Vogl, W. Weise, *Prog. Part. Nucl. Phys.* **27** (1991) 195.

[7] S. Klevansky, *Rev. Mod. Phys.* **64** (1992) 649.

[8] T. Hatsuda, T. Kunihiro, *Phys. Rep.* **247** (1994) 221 [hep-ph/9401310].

[9] D. Ebert, H. Reinhardt, M.K. Volkov, *Prog. Part. Nucl. Phys.* **33** (1994) 1.

[10] J. Bijnens, *Phys. Rep.* **265** (1996) 369 [hep-ph/9502335].

[11] A.A. Andrianov, V.A. Andrianov, *Int. J. Mod. Phys.* **A 8** (1993) 1981; *Nucl. Phys.* **39BC** *(Proc. Suppl.)* (1995) 257.

[12] E. Pallante, R. Petronzio, *Z. Physik* **C 65** (1995) 487 [hep-ph/9401305].

[13] A.A. Andrianov, V.A. Andrianov, V. L. Yudichev, *Theor. Math. Phys.* **108** (1996) 1069.

[14] J. Gasser, H. Leutwyler, *Nucl. Phys.* **B 250** (1985) 465.

[15] G. Ecker, *Prog. Part. Nucl. Phys.* **35** (1995) 1 [hep-ph/9501357].

[16] A. Pich, *Rept. Prog. Phys.* **58** (1995) 563.

[17] J.F. Donoghue, C. Ramirez, G. Valencia, *Phys. Rev.* **D 39** (1989) 1947.

[18] G. Ecker, J. Gasser, H. Leutwyler, A. Pich, E. de Rafael, *Phys. Lett.* **B 223** (1989) 425.

[19] A.A. Andrianov, V.A. Andrianov, A.N. Manashov, *Int. J. Mod. Phys.* **A 6** (1991) 5435.

[20] A.A. Andrianov, V.A. Andrianov, *Phys. Atom. Nucl.* **56** (1993) 855 (*Yad. Fiz.* **56** (1993) 249).

[21] A.A. Andrianov, *Phys. Lett.* **B 157** (1985) 425.

[22] D. Dyakonov, V. Petrov, *Nucl. Phys.* **B 272** (1986) 457.

[23] A.A. Andrianov, V.A. Andrianov, V.Yu. Novozhilov, Yu.V. Novozhilov, *Lett. Math. Phys.* **11** (1986) 217; *Theor. Math. Phys.* **70** (1987) 43; *Phys. Lett.* **B 186** (1987) 401.

[24] A.A. Andrianov, V.A. Andrianov, D. Ebert, T. Feldmann, *Int. J. Mod. Phys.* **A 12** (1997) 5589 [hep-ph/9702338].

[25] J. Bijnens, C. Bruno, E. de Rafael, *Nucl. Phys.* **B 390** (1993) 501 [hep-ph/9206236].

[26] J. Bijnens, E. de Rafael, H. Zheng, *Z. Physik* **C 62** (1994) 437 [hep-ph/9306323].

[27] D. Ebert, A.A. Bel'kov, A.V. Lanyov, A. Schaale, *Int. J. Mod. Phys.* **A 8** (1993) 1313.

[28] R.D. Ball, *Phys. Rep.* **182** (1989) 1.

[29] M.A. Shifman, A.I. Vainstein, V.I. Zakharov. *Nucl. Phys.* **B 147** (1979) 385; *Nucl. Phys.* **B 147** (1979) 448.

[30] L.J. Reinders, H. Rubinstein, S. Yazaki, *Phys. Rep.* **127** (1985) 1 and references therein.

[31] M. Jamin, M. Münz, *Z. Physik* **C 66** (1995) 633 [hep-ph/9409335].

[32] B. Moussallam, J. Stern, *Chiral symmetry aspects of the scalars*, hep-ph/9404353.

[33] A.A. Andrianov, V.A. Andrianov, *Zap. Nauchn. Semin. LOMI*, **245** (1996) 5 [hep-ph/9705364].

[34] Particle Data Group, *Eur. Phys. J.* **C 3** (1998) 1.

[35] M. Knecht, E. de Rafael, *Phys. Lett.* **B 424** (1998) 335 [hep-ph/9712457].

[36] S. Peris, M. Perrottet, E. de Rafael, *J. High Energy Phys.* **05** (1998) 011 [hep-ph/9805442].

[37] M. Knecht, S. Peris, E. de Rafael, *Phys. Lett.* **B 443** (1998) 255 [hep-ph/9809594].

[38] J.F. Donoghue, E. Golowich, *Phys. Rev.* **D 49** (1994) 1513 [hep-ph/9307262].

[39] B. Moussallam, *Nucl. Phys.* **B 504** (1997) 381 [hep-ph/9701400].

[40] G. t' Hooft, *Nucl. Phys.* **B 72** (1974) 461.

[41] E. Witten, *Nucl. Phys.* **B 160** (1979) 57.

[42] S. Weinberg, *Phys. Rev. Lett.* **18** (1967) 507.

[43] H.G. Dosch, S. Narison, *Phys. Lett.* **B 417** (1998) 173 [hep-ph/9709215].

[44] G. Ecker, J. Gasser, A. Pich, E. de Rafael, *Nucl. Phys.* **B 321** (1989) 311.

[45] C. Dominguez, *Phys. Rev.* **D 16** (1977) 2313; 2320.

[46] A.L. Kataev, N.V. Krasnikov, A.A. Pivovarov, *Phys. Lett.* **B 123** (1983) 93.

[47] V. Elias, A. Fariborz, M.A. Samuel, Fang Shi, T.G. Steele, *Phys. Lett.* **B 412** (1997) 131 [hep-ph/9706472].

[48] T. Hannah, *Phys. Rev.* **D 54** (1996) 4648 [hep-ph/9611307].

[49] J.A. Oller, E. Oset, *Phys. Rev.* **D 60** (1999) 074023 [hep-ph/9809337].

JHEP10(1999)022